

## 5. DIVIDE AND CONQUER I

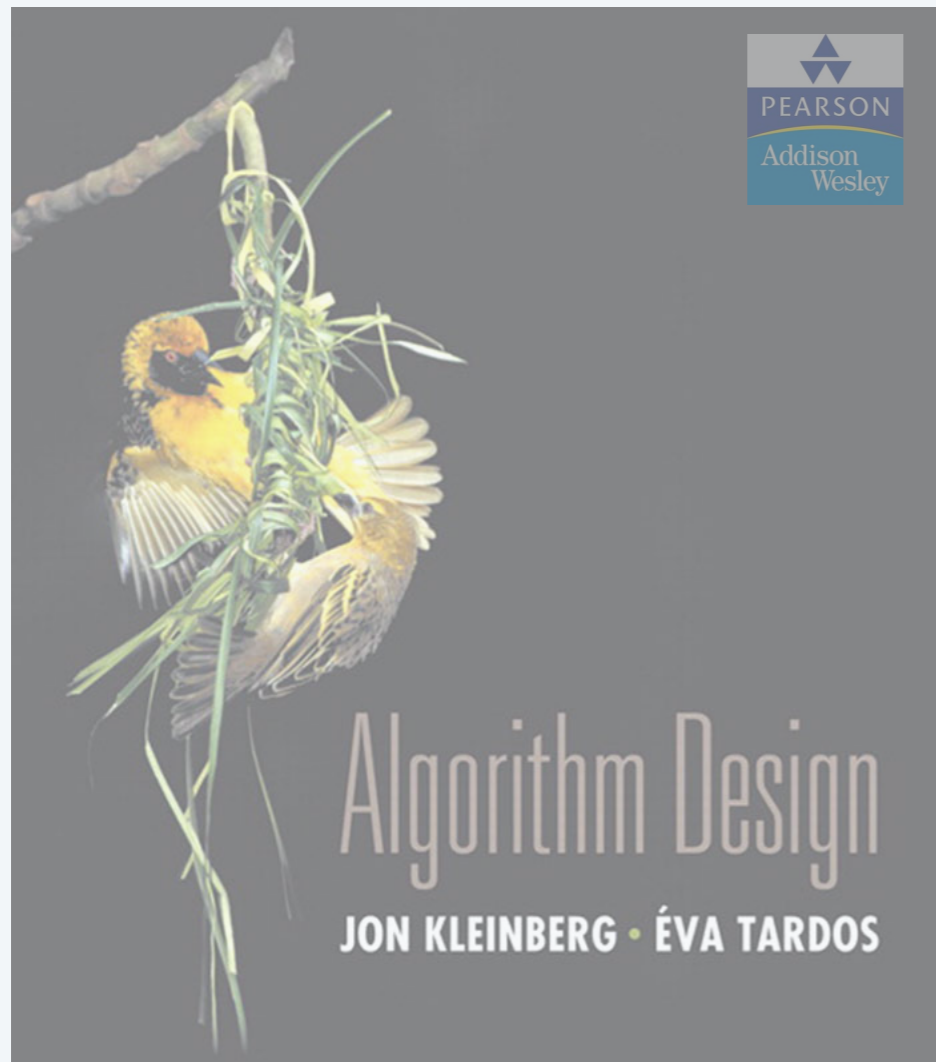
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- ▶ *merge demo*
- ▶ *merge-and-count demo*

Lecture slides by Kevin Wayne

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<http://www.cs.princeton.edu/~wayne/kleinberg-tardos>



SECTIONS 5.1–5.2

## 5. DIVIDE AND CONQUER

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- ▶ *merge demo*
- ▶ *merge-and-count demo*

# Merge demo

---

Given two sorted lists  $A$  and  $B$ , merge into sorted list  $C$ .

sorted list A

3	7	10	14	18
---	---	----	----	----

sorted list B

2	11	16	20	23
---	----	----	----	----

# Merge demo

---

Given two sorted lists  $A$  and  $B$ , merge into sorted list  $C$ .

sorted list A

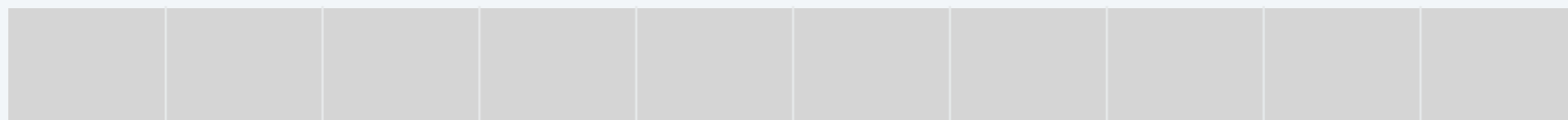


sorted list B



compare minimum entry in each list: copy 2

sorted list C



# Merge demo

---

Given two sorted lists  $A$  and  $B$ , merge into sorted list  $C$ .

sorted list A

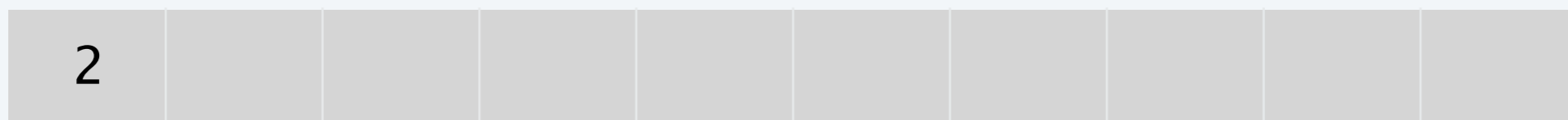


sorted list B



compare minimum entry in each list: copy 3

sorted list C



# Merge demo

---

Given two sorted lists  $A$  and  $B$ , merge into sorted list  $C$ .

sorted list A

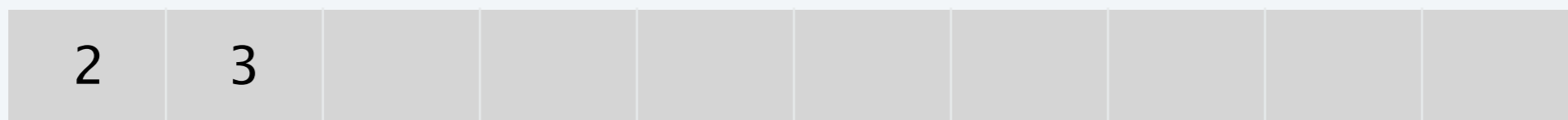


sorted list B



compare minimum entry in each list: copy 7

sorted list C



# Merge demo

---

Given two sorted lists  $A$  and  $B$ , merge into sorted list  $C$ .

sorted list A

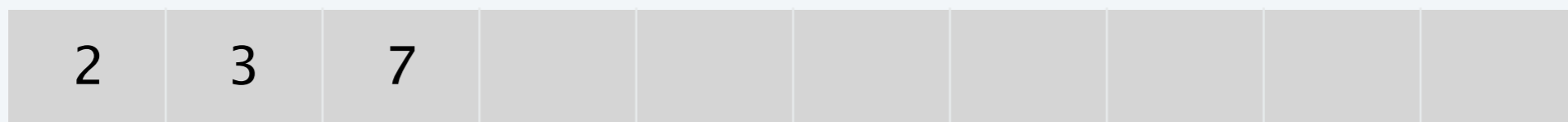


sorted list B



compare minimum entry in each list: copy 10

sorted list C



# Merge demo

---

Given two sorted lists  $A$  and  $B$ , merge into sorted list  $C$ .

sorted list A

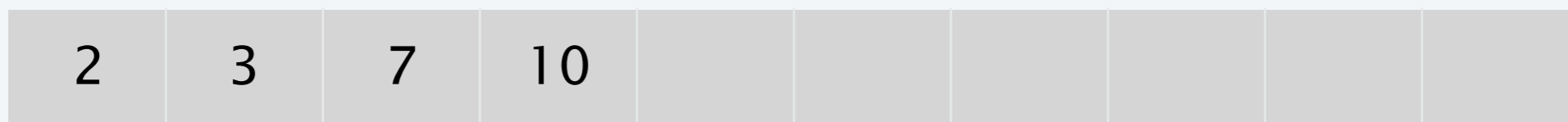


sorted list B



compare minimum entry in each list: copy 11

sorted list C





# Merge demo

---

Given two sorted lists  $A$  and  $B$ , merge into sorted list  $C$ .

sorted list A



sorted list B



compare minimum entry in each list: copy 14

sorted list C



# Merge demo

---

Given two sorted lists  $A$  and  $B$ , merge into sorted list  $C$ .

sorted list A



sorted list B



compare minimum entry in each list: copy 16

sorted list C



# Merge demo

---

Given two sorted lists  $A$  and  $B$ , merge into sorted list  $C$ .

sorted list A



sorted list B



compare minimum entry in each list: copy 18

sorted list C



# Merge demo

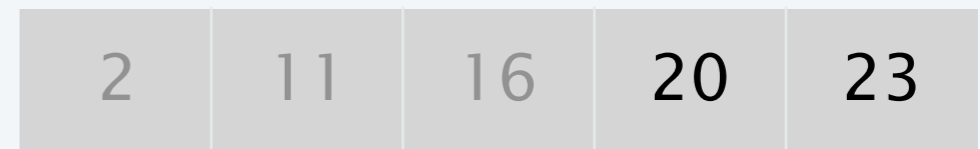
---

Given two sorted lists  $A$  and  $B$ , merge into sorted list  $C$ .

sorted list A



sorted list B



list A exhausted: copy 20

sorted list C



# Merge demo

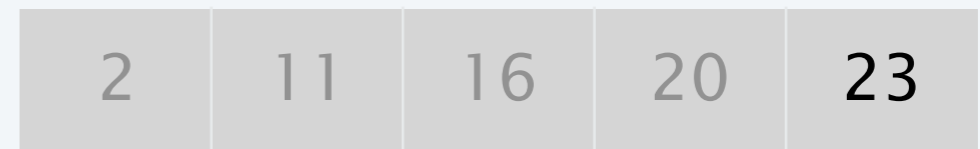
---

Given two sorted lists  $A$  and  $B$ , merge into sorted list  $C$ .

sorted list A



sorted list B



list A exhausted: copy 23

sorted list C



# Merge demo

---

Given two sorted lists  $A$  and  $B$ , merge into sorted list  $C$ .

sorted list A



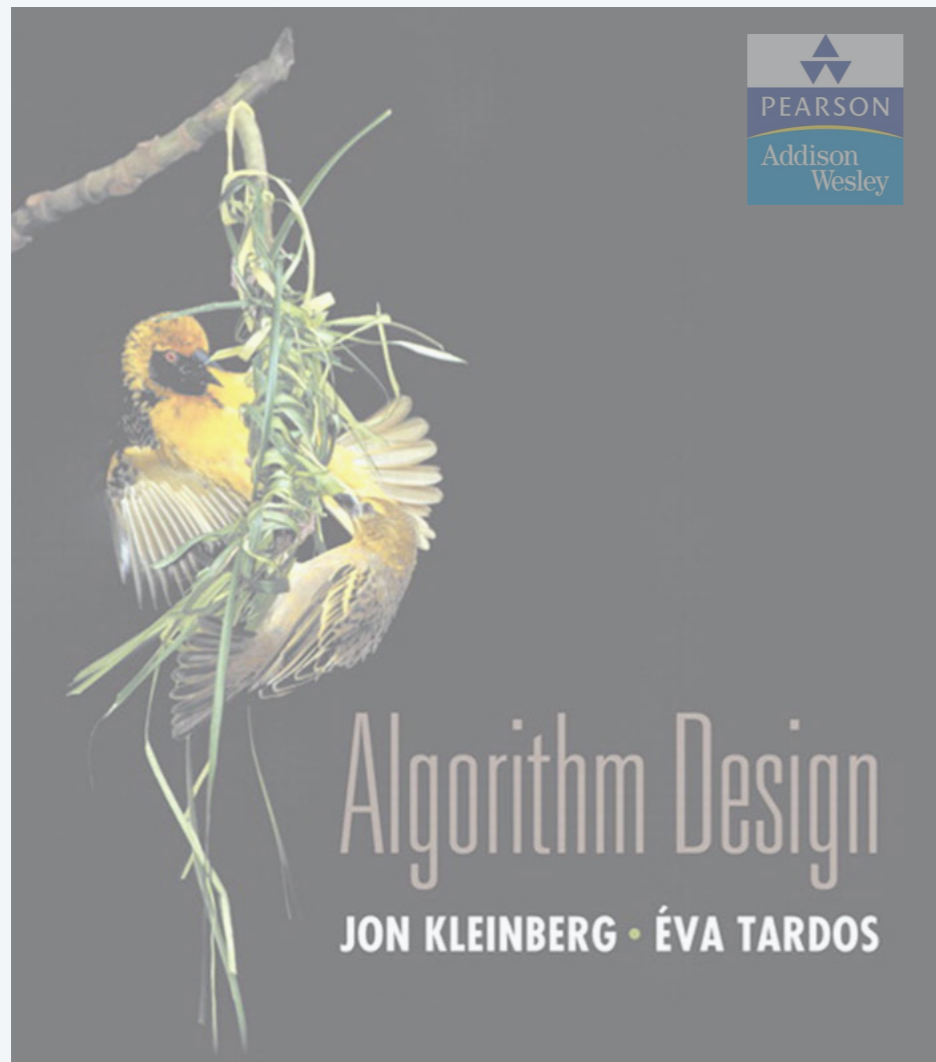
sorted list B



done

sorted list C





## SECTION 5.3

# 5. DIVIDE AND CONQUER

---

- ▶ *merge demo*
- ▶ *merge-and-count demo*

# Merge-and-count demo

---

Given two sorted lists  $A$  and  $B$ ,

- Count number of inversions  $(a, b)$  with  $a \in A$  and  $b \in B$ .
- Merge  $A$  and  $B$  into sorted list  $C$ .

sorted list A

3	7	10	14	18
---	---	----	----	----

sorted list B

2	11	16	20	23
---	----	----	----	----



# Merge-and-count demo

---

Given two sorted lists  $A$  and  $B$ ,

- Count number of inversions  $(a, b)$  with  $a \in A$  and  $b \in B$ .
- Merge  $A$  and  $B$  into sorted list  $C$ .

sorted list A



sorted list B



compare minimum entry in each list: copy 2 and add  $x$  to inversion count

sorted list C



$x = 5$  ← number of elements remaining in  $A$   
inversions = 0

# Merge-and-count demo

---

Given two sorted lists  $A$  and  $B$ ,

- Count number of inversions  $(a, b)$  with  $a \in A$  and  $b \in B$ .
- Merge  $A$  and  $B$  into sorted list  $C$ .

sorted list A



sorted list B



5



compare minimum entry in each list: copy 3 and decrement  $x$

sorted list C



$x = 5$

inversions = 5

# Merge-and-count demo

---

Given two sorted lists  $A$  and  $B$ ,

- Count number of inversions  $(a, b)$  with  $a \in A$  and  $b \in B$ .
- Merge  $A$  and  $B$  into sorted list  $C$ .

sorted list A



sorted list B

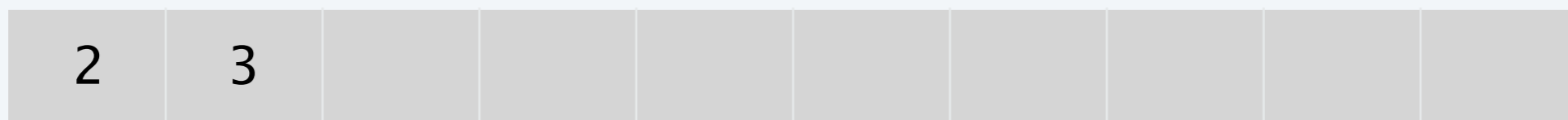


5



compare minimum entry in each list: copy 7 and decrement  $x$

sorted list C



$x = 4$

inversions = 5

# Merge-and-count demo

---

Given two sorted lists  $A$  and  $B$ ,

- Count number of inversions  $(a, b)$  with  $a \in A$  and  $b \in B$ .
- Merge  $A$  and  $B$  into sorted list  $C$ .

sorted list A



sorted list B

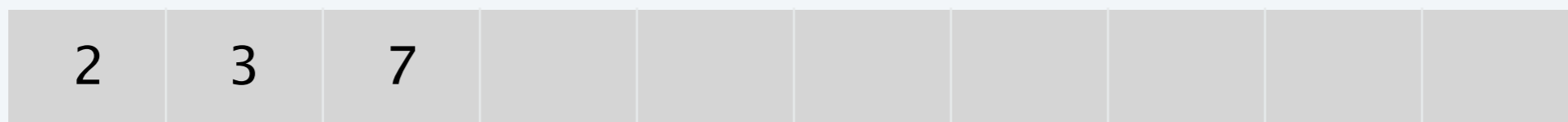


5



compare minimum entry in each list: copy 10 and decrement  $x$

sorted list C



$x = 3$

inversions = 5

# Merge-and-count demo

---

Given two sorted lists  $A$  and  $B$ ,

- Count number of inversions  $(a, b)$  with  $a \in A$  and  $b \in B$ .
- Merge  $A$  and  $B$  into sorted list  $C$ .

sorted list A



sorted list B

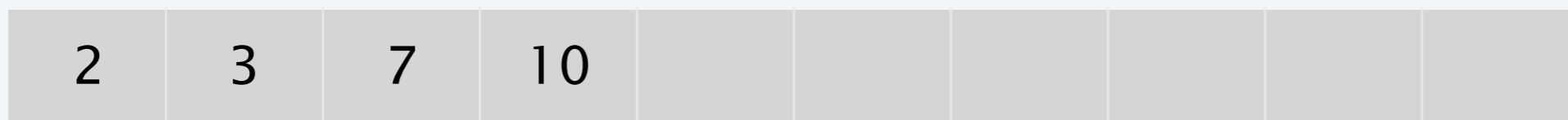


5



compare minimum entry in each list: copy 11 and add x to increment count

sorted list C



$x = 2$

**inversions = 5**

# Merge-and-count demo

---

Given two sorted lists  $A$  and  $B$ ,

- Count number of inversions  $(a, b)$  with  $a \in A$  and  $b \in B$ .
- Merge  $A$  and  $B$  into sorted list  $C$ .

sorted list A



sorted list B



5

2



compare minimum entry in each list: copy 14 and decrement  $x$

sorted list C



$x = 2$

inversions = 7

# Merge-and-count demo

---

Given two sorted lists  $A$  and  $B$ ,

- Count number of inversions  $(a, b)$  with  $a \in A$  and  $b \in B$ .
- Merge  $A$  and  $B$  into sorted list  $C$ .

sorted list A



sorted list B



5

2



compare minimum entry in each list: copy 16 and add  $x$  to increment count

sorted list C



$x = 1$

inversions = 7

# Merge-and-count demo

---

Given two sorted lists  $A$  and  $B$ ,

- Count number of inversions  $(a, b)$  with  $a \in A$  and  $b \in B$ .
- Merge  $A$  and  $B$  into sorted list  $C$ .

sorted list A



sorted list B



5

2

1

↑

compare minimum entry in each list: copy 18 and decrement  $x$

sorted list C



$x = 1$

inversions = 8



# Merge-and-count demo

---

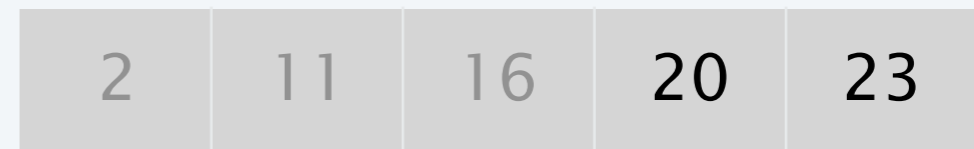
Given two sorted lists  $A$  and  $B$ ,

- Count number of inversions  $(a, b)$  with  $a \in A$  and  $b \in B$ .
- Merge  $A$  and  $B$  into sorted list  $C$ .

sorted list A



sorted list B



5

2

1



list A exhausted: copy 20

sorted list C



$x = 0$

inversions = 8

# Merge-and-count demo

---

Given two sorted lists  $A$  and  $B$ ,

- Count number of inversions  $(a, b)$  with  $a \in A$  and  $b \in B$ .
- Merge  $A$  and  $B$  into sorted list  $C$ .

sorted list A



sorted list B



5

2

1

0

↑

list A exhausted: copy 23

sorted list C



$x = 0$

inversions = 8

# Merge-and-count demo

---

Given two sorted lists  $A$  and  $B$ ,

- Count number of inversions  $(a, b)$  with  $a \in A$  and  $b \in B$ .
- Merge  $A$  and  $B$  into sorted list  $C$ .

sorted list A



sorted list B



5

2

1

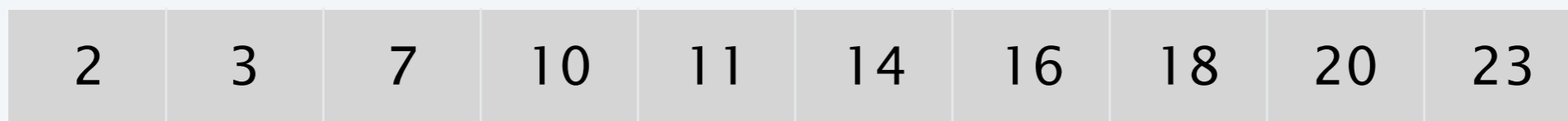
0

0



**done: return 8 inversions**

sorted list C



$x = 0$

**inversions = 8**